

PROCESSING SYMBOLIC NUMERICAL INFORMATION AND ITS IMPLICATIONS FOR MATHEMATICS LEARNING

EDITED BY: Ricardo Moura, Julia Bahnmüller, Vitor Geraldi Haase,
Júlia Beatriz Lopes-Silva and Korbinian Moeller
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PROCESSING SYMBOLIC NUMERICAL INFORMATION AND ITS IMPLICATIONS FOR MATHEMATICS LEARNING

Topic Editors:

Ricardo Moura, University of Brasilia, Brazil

Julia Bahnmueller, Loughborough University, United Kingdom

Vitor Geraldi Haase, Federal University of Minas Gerais, Brazil

Júlia Beatriz Lopes-Silva, Federal University of Minas Gerais, Brazil

Korbinian Moeller, Loughborough University, United Kingdom

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Editorial: Processing Symbolic Numerical Information and Its Implications for Mathematics Learning

Ricardo Moura^{1*}, Julia Bahnmueller², Vitor Geraldi Haase³, Júlia Beatriz Lopes-Silva³ and Korbinian Moeller²

¹ Institute of Psychology, University of Brasília, Brasília, Brazil, ² Department of Mathematics Education, Loughborough University, Loughborough, United Kingdom, ³ Department of Psychology, Federal University of Minas Gerais, Belo Horizonte, Brazil

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Editorial on the Research Topic

Processing Symbolic Numerical Information and Its Implications for Mathematics Learning

INTRODUCTION

Among the first demands of enculturation on children's emerging numerical abilities is learning to transcode their initial representations of quantity into symbolic notations. It usually takes years of (formal) education before children master symbolic numerical representations including number words and digital-Arabic numerals as well as their place-value structuring and apply this successfully to perform arithmetic.

Understanding how symbolic and non-symbolic numerical representations relate to each other as well as predict numerical development may help to elucidate how children acquire numerical skills including arithmetic. In particular, as there is evidence suggesting that mastery of symbolic numerical representations—more so than non-symbolic ones—are building blocks for later arithmetic / mathematics performance.

The present Research Topic aimed at providing latest research results on processing of symbolic numerical information but also the trajectories in which processing of both symbolic and non-symbolic numerical information impacts numerical development. We collated 14 empirical studies focusing on (i) the semantic processing of symbolic numbers, (ii) the processing of symbolic numbers as a predictor of arithmetic / mathematics performance, and (iii) the understanding of the place-value structuring of symbolic numbers. These will be discussed in turn in the following.

SEMANTIC PROCESSING OF SYMBOLIC NUMBERS

One major question when it comes to the processing of number symbols is how they become associated with and under what circumstances they activate semantic magnitude information. The latter was investigated by Malykh et al. who observed that accuracy and speed of processing non-symbolic magnitudes developed differently from 1st to 9th grade, which might suggest different dependent variables reflect different underlying processes.

Finke et al. investigated cross-format activation of Arabic digits and number words in two ERP experiments with adult participants. They observed that number pairs seemed to be processed in two stages. At an early stage, number pairs presented in the same notation are integrated

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Douglas F. Kauffman,
Medical University of the Americas –
Nevis, United States

*Correspondence:

Ricardo Moura
ricardomoura@unb.br

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automatically without necessary access to semantic magnitude information. However, the latter is involved in a later second stage supporting cross-format integration of numbers.

In a further ERP study, van Hoogmoed et al. investigated the integration of numerical information presented in symbolic (Arabic digits) and non-symbolic (dot patterns) presentation formats. In line with the results of Finke et al.—but in contrast to ideas of an approximate number system underlying human numerical cognition—their results did not support the idea that children automatically activate semantic magnitude information when processing symbolic numbers.

When learning to understand symbolic numbers, a main challenge faced by children is to grasp the concept of the number zero and how to use this number in different numerical tasks and notations. In this context, Krajcsi et al. demonstrated that children understand verbal labels reflecting zero (e.g., nothing) and deal with empty sets even before they regard zero as a number.

Furthermore, Schmidt et al. investigated how early neuromotor experiences influence spatial associations for symbolic numbers. The authors considered two neuromuscular diseases characterized by progressive loss of motor abilities: spinal muscular atrophy (SMA, preventing any experiences of independent motoric exploration) and Duchenne muscular dystrophy (DMD, which compromises acquired experiences later in development). Results indicated that children with DMD exhibited typical spatial associations when processing symbolic numbers, while children with SMA exhibited no such or even reversed associations. These results corroborate the relevance of early sensorimotor experiences for children's numerical development.

When looking at numerical development in particular, it often is of specific interest if, and if so, which basic numerical skills—including the processing of symbolic numbers—predict later arithmetic / mathematics performance in what way. This question was addressed in another set of studies of our Research Topic.

PROCESSING SYMBOLIC NUMBERS AS A PREDICTOR OF ARITHMETIC / MATHEMATICS PERFORMANCE

It has been argued that mathematics achievement builds on more basic numerical skills with inconsistent findings regarding the relevance of processing non-symbolic vs. symbolic numerical information. In this context, Gloor et al. provided evidence that both Spontaneous Focusing on Numerosity as well as symbolic number skills longitudinally predicted mathematics achievement at the end of 1st grade. However, the contribution of symbolic numerical skills was observed to be more pronounced.

Furthermore, Chen et al. studied a sample of adolescents with congenital and acquired deafness to investigate associations between symbolic and non-symbolic magnitude processing and arithmetic performance. They observed significant associations between symbolic and non-symbolic processing and arithmetic

performance even after controlling for demographic variables. Interestingly, however, number magnitude processing did not predict arithmetic performance in the group with congenital deafness indicating an influence of (hearing) language on numerical development.

Additionally, Räsänen et al. report on a cross-sectional large-scale study investigating the development of basic number processing skills (i.e., single-digit symbolic number comparison, digit-dot matching) and arithmetic fluency in children aged 9–15 years using data from the development of a screening tool for mathematics learning disabilities. They observed that girls performed better in tasks on basic number processing whereas boys performed better in tasks related to arithmetic fluency. This implies that gender should be considered when assessing mathematical learning disabilities.

Moreover, Abreu-Mendoza et al. evaluated the effectiveness of an intervention fostering non-symbolic proportional reasoning by means of Cuisenaire rods for improving symbolic fraction knowledge. Results indicated that the intervention significantly improved processing of non-symbolic continuous proportions, but did not significantly improve processing of discretized proportions such as symbolic fractions.

Furthermore, Vogel et al. investigated the association of mathematical abilities and the reversed numerical distance effect typically observed in order judgments of number sequences [i.e., faster RTs for sequences with small (2 3 4) compared to large distances (2 4 6)]. In an adult sample, they observed that not all individuals presented a significant reversed distance effect and that the association of order judgements with mathematical abilities was more pronounced for individuals who exhibited a significant reversed distance effect.

Finally, Loenneker et al. investigated the association of visuo-spatial and symbolic arithmetical skills in a sample of patients with Parkinson's Disease (PD) in different stages of the disease. Results indicated that the occurrence of arithmetic difficulties was predicted by attentional and visuo-spatial/constructional deficits even after controlling for clinical and sociodemographic confounds. Interestingly, patients' difficulties were mostly related to place value processing in calculation tasks, which highlights the relevance of evaluating this basic numerical skill in neuropsychological patients.

Importantly, place-value processing—reflecting knowledge of the structuring principle of symbolic numbers—is critical to numerical development. This was addressed in a final set of studies included in the Research Topic.

PLACE-VALUE UNDERSTANDING

Current research indicates that understanding the place-value structure of the Arabic number system is a challenge for children during the 1st years of primary school (e.g., Moura et al., 2015). At the same time, it is crucial for later mathematics achievement (e.g., Moeller et al., 2011). However, research into the theoretical underpinnings and developmental trajectories of place-value understanding as well as effective interventional approaches is currently limited.

Addressing these gaps, Herzog and Fritz-Stratmann link their recently proposed hierarchical model of place-value learning (Herzog et al., 2019) to number transcoding (i.e., writing numbers to dictation or reading digital-Arabic numbers aloud)—a commonly employed task used to index place-value understanding. The authors found that transcoding may indeed be a valid index for place-value understanding because 2nd and 3rd graders demonstrating more advanced levels of place-value understanding also performed better in writing Arabic numbers to dictation, especially for syntactically more complex numbers (e.g., including zeros).

Friedmann et al. further contribute to our theoretical knowledge of place-value understanding by presenting a single case study on the performance of a deaf participant who also presented specific deficits in transcoding multi-digit numbers. While word reading seemed to be preserved, reading and comprehension of multi-digit Arabic numbers was impaired with error patterns reflecting characteristics of the sign language used by the participant. The authors interpreted this as evidence for dissociated mechanisms for processing visual properties and the decimal structure of Arabic numbers.

Providing further insights regarding interventional approaches to foster place-value understanding, Yuan et al. evaluated in three studies whether and if so how children aged 4–6 years that had not yet entered school benefited from different approaches specifically designed to foster conceptual

understanding of place-value in general and transcoding (i.e., reading multi-digit Arabic numbers) in particular. Based on their findings the authors argue that rather than traditional mathematical manipulatives (e.g., base-10-blocks, Abacus), shapes with a simple but easily graspable structure, or even exposure to pairs of multidigit Arabic numbers and their names, may be more effective for the acquisition of initial place-value understanding.

CONCLUDING REMARKS

As indicated by the wide range of topics addressed in this Research Topic, the relevance of symbolic processing for numerical cognition and its development is undeniable. However, even though it seems a very basic numerical skill, it nevertheless poses considerable challenges on learners especially with respect to acquiring understanding of the organizing place-value structuring principle of symbolic Arabic numbers. Taken together, this Research Topic indicates that understanding symbolic numbers (including their place-value structuring) seems key to successful numerical development.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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